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Quantum Effects of Mesoscopic *RLC* Circuit in Squeezed Vacuum State

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Starting from the equation of motion of an active *RLC* circuit, the quantum effects of charge and current in the mesoscopic circuit (*RLC* circuit) in the squeezed vacuum state are investigated.

1. INTRODUCTION

The rapid development of nanoelectronics and nanometer-scale technology (Srivastava and Widom, 1987; Buot, 1993) has led to a strong trend in the miniaturization of integrated circuits and components toward atomic-scale dimensions (Garcia, 1992). Clearly, the quantum effects of the electronic devices and circuits must be taken into account when the transport length reaches a characteristic dimension, namely, when the charge-carrier inelastic coherence length approaches the Fermi wavelength. Louisell (1973) discussed the quantum effect of an *LC* circuit in the vacuum state and gave the quantum noise in the circuit. Recently, Chen *et al.* (1995) investigated the quantum effects of charge and current in a mesoscopic *RLC* circuit with source in the vacuum state. However, the quantum effects of charge and current of the circuit in the squeezed vacuum state have not been reported. As the vacuum state can be regarded as a special case of the squeezed vacuum state when the squeeze parameter is zero, the study of the quantum effects of charge and current in the squeezed vacuum state of a mesoscopic circuit would be more general and significant. In this paper, we discuss the quantum effects of charge and current of the *RLC* circuit in the squeezed vacuum state.

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2. QUANTUM EFFECTS OF CHARGE AND CURRENT IN A MESOSCOPIC *RLC* CIRCUIT IN THE SQUEEZED VACUUM STATE

The equation of motion for an *RLC* circuit with source is

$$\frac{d^2q(t)}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \omega_0^2 q(t) = \frac{1}{L}\,\varepsilon(t) \tag{1}$$

where L, R, and C stand for inductance, resistance, and capacity, respectively, $\omega_0^2 = 1/(LC)$ is the resonance frequency of the LC circuit without the resistance R, $q(t)$ is the charge, and $\varepsilon(t)$ is the electromotive force.

The Hamiltonian of an *RLC* circuit with source after quantization (Chen *et al.*, 1995) is

$$H = \hbar(\omega - i\lambda)a^*a + a^*S(t) + aS^*(t) \tag{2}$$

where $\omega^2 = \omega_0^2 - \lambda^2$, $\lambda = R/(2L)$, and $S(t) = i\sqrt{\hbar/2}\;\varepsilon(t)/L$. Here a and a^* are the generalized lowering and raising operators, respectively:

$$a = (2\hbar\omega)^{-1/2}[j + (\lambda - i\omega)q] \tag{3}$$

$$a^* = (2\hbar\omega)^{-1/2}[j + (\lambda + i\omega)q] \tag{4}$$

and they satisfy the commutation relation $[a, a^*] = 1$.

When the circuit has no power, $S(t) = 0$, we assume that the circuit is in the squeezed vacuum state (Fan and Guo, 1985)

$$|0\rangle_r = \operatorname{sech}^{1/2} r \sum_{n=0}^{\infty} \frac{(-\tanh r)^n[(2n)!]^{1/2}}{n!\;2^n}\,|2n\rangle \tag{5}$$

where r is squeeze parameter ($0 < r < \infty$). From (3)–(5), the mean values and mean square values of charge and current are, respectively,

$$\overline{q} = {}_r\langle 0|q|0\rangle_r = 0 \tag{6}$$

$$\overline{j} = {}_r\langle 0|j|0\rangle_r = 0 \tag{7}$$

$$\begin{aligned}\overline{q^2} &= {}_r\langle 0|q^2|0\rangle_r \\ &= \frac{\hbar}{2\omega}\operatorname{sech} r \sum_{n=0}^{\infty} \frac{(2n)!\,[1 + 4n + 2(2n+1)\tanh r]\tanh^{2n} r}{(n!)^2\,2^{2n}}\end{aligned} \tag{8}$$

$$\begin{aligned}\overline{j^2} &= {}_r\langle 0|j^2|0\rangle_r \\ &= \frac{\hbar\omega_0^2}{2\omega}\operatorname{sech} r \sum_{n=0}^{\infty} \frac{(2n)!\,[1 + 4n + 2(2n+1)\tanh r]\tanh^{2n} r}{(n!)^2\,2^{2n}}\end{aligned}$$

$$+ \frac{\hbar\omega}{2} \operatorname{sech} r \sum_{n=0}^{\infty} \frac{(2n)!\, 4(2n+1)(-\tanh r)^{2n+1}}{(n!)^2\, 2^{2n}} \tag{9}$$

Therefore, in the squeezed vacuum state, the mean values of charge and current are zero in an *RLC* circuit without source after quantization, but the mean-squared results are not. Therefore, there are quantum fluctuations of charge and current in an *RLC* circuit without source in the squeezed vacuum state. It is evident that the product of the quantum fluctuations of charge and current are related to the electronic device and squeeze parameter r.

Particularly, when $r = 0$, from (6)–(9), we have

$$\overline{(\Delta q)^2}\,\overline{(\Delta j)^2} = \frac{\hbar^2}{4} \frac{4L}{4L - R^2C} \tag{10}$$

This is the immediate result of the commutative relation $[q, j] = i\hbar$, and gives the quantum effect in the circuit, i.e., the uncertainty relation. From equation (10), it can be seen that the quantum fluctuations of charge and current in an *RLC* circuit are interrelated. There exists a squeezing effect between them. An increase of the quantum noise of the charge will result in a decrease of the quantum noise of the current, and vice versa.

3. CONCLUSIONS

From the discussion we conclude that, for an *RLC* circuit with power, the quantum fluctuations of charge and current can also emerge in the squeezed vacuum state, and hence are analogous to those in a normal vacuum state. We believe that these effects in the squeezed vacuum state will also exist in other mesoscopic circuits (we shall discuss them elsewhere). This is a macroscopic quantum effect and will definitely be significant in the design of microcircuits. Moreover, as the vacuum state is the squeezed vacuum state with the squeeze parameter $r = 0$, the quantum effect of the *RLC* circuit discussed in Chen *et al.* (1995) is only a special case of our general conclusion when $r = 0$.

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